VI. Standards and Thinking Strategies in Mathematical Problem Solving

Mathematicians explore and investigate an often uncertain world and attempt to explain what they see. Sometimes a single explanation works; more often, patterns, and relationships emerge that lead to several different conclusions. Logic helps mathematicians justify their methods and determine the validity of their solutions as they try to make sense of the unknown. PEBC's professional development in mathematics aligns with the Colorado Model Content Standards in Mathematics and the National Council of Teachers of Mathematics, which focus on authentic problem-solving and clear communication about mathematical thinking and processes.

Application of mathematical knowledge in problem-solving situations
(All Colorado Model Content Standards, NCTM Standard 6)
We demonstrate techniques that enhance student problem-solving ability. Teachers learn to teach problem solving and use a recursive framework to help students become proficient problem solvers.

Communication of reasoning
(All Colorado Model Content Standards, NCTM Standard 8)
We help teachers develop questioning techniques that not only probe student thinking, but also require students to explain their thinking and to justify their answers orally and in writing.

Development of meaningful mathematical knowledge
(All Colorado Model Content Standards, NCTM Standards 1-5)
We help teachers increase their own understanding of the math content – number sense, patterns, algebra, data analysis including statistics and probability, geometry, measurement, and computation. Staff developers model lessons that focus on conceptual understanding and the meaningful development of mathematical procedures. We also give teachers research to help them understand how they can help their students make sense of mathematics.

Below we highlight how students use specific thinking strategies in Mathematical Problem Solving.[1]

Thinking Strategy	Cognitive Behaviors
Monitoring for Meaning	• Mathematicians are metacognitive as they continually ask themselves, "Does this make sense?" and "Is my answer reasonable?" • Mathematicians use accurate math vocabulary and show their work in clear concise forms so others can follow their thinking without asking questions.
Activating, Using and Building Background Knowledge (schema)	• Use their prior knowledge to generalize about similar problems and to choose problem solving strategies. • Mathematicians add to schema by trying more challenging problems and hearing for others about different problem solving methods.
Asking Questions	• Mathematicians test theories/answers/hypotheses by asking questions about various approaches to a problem. • Mathematicians extend their own thinking by asking themselves questions for which they don't have answers.
Drawing Inferences	• Mathematicians use patterns and relationships to generalize and infer what come next in the problem-solving process. • Mathematicians solve problems in different ways and support their methods through proof, number sentences, pictures, charts, and graphs.
Determining Importance	• Mathematicians gather text information from graphs, charts, and tables. • Mathematicians use key words to decide what information is relevant and irrelevant to a problem.
Creating Sensory Images	• Mathematicians use mental pictures/models of shapes, numbers, and processes to build an understanding of concepts and problems and to experiment with ideas. • Mathematicians visually represent their thinking through drawings, pictures, graphs, models, and charts.
Synthesizing Information	• Mathematicians generalize from patterns they observe. • Mathematicians synthesize math concepts when they use them in real-life applications.

[1] Adapted by Pearson/Dole revised Tovani & James Donouhue.